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# Bringing Data to Life with Post-Hoc CUSUM Charts

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*In analyzing healthcare data and presenting the results to clinical and management colleagues the authors have found that both Shewart and CUSUM charts have been invaluable. In particular the post-hoc CUSUM chart is a useful tool for the detection of improvement in the level of performance of healthcare processes. The charts of the healthcare data displayed and discussed were created using the widely available statistical software package Minitab but information on how to create the charts using Microsoft Excel is included. The authors believe that this case study would be of value to classes in applied statistics at intermediate level in courses including an element on applications to process improvement.*

## Introduction

W. A. Shewhart, an employee of the Western Electric Company in the USA during the 1920s, first proposed the use of control charts. His contribution in this field is so important that the charts he developed are often referred to as “Shewhart charts”. In the early 1950s E. S. Page in the UK proposed an alternative type of control chart known as the cumulative sum (CUSUM) chart. Both types of chart are widely used not just in manufacturing but also in other areas such as healthcare. In performing retrospective analysis of data on stroke patient care, hospital-acquired methicillin resistant *Staphylococcus aureus* (MRSA) infections and joint replacement surgery patient outcomes the authors have found both Shewhart and CUSUM charts of value in addressing the

fundamental question “How do you know that your care is improving?” posed by Raymond G Carey (2002). In particular the use of post-hoc CUSUM charts has proved particularly informative.

Many people are unfamiliar with CUSUM charts so by way of introduction imagine a work cell where the target production rate is 80 units per week. Each week the cell manager agrees to put £1 in the Christmas party fund for every unit produced over target and to remove £1 from the fund for every unit the cell is below target. Finally consider the scheme to start at the beginning of 2009 with the fund empty. Data for a 20-week period is shown in Table 1.

Table 1.

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Units Produced	90	90	90	90	90	90	90	80	80	80	80	80	75	75	75	75	75	75	75	75
Deviation from target	10	10	10	10	10	10	10	0	0	0	0	0	-5	-5	-5	-5	-5	-5	-5	-5
Cumulative sum in fund	10	20	30	40	50	60	70	70	70	70	70	70	65	60	55	50	45	40	35	30

Table 2.

Observation	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Viscosity	Xi	9.1	9.2	9.3	8.8	8.5	9.1	9.1	8.8	8.8	8.6	8.9	9.3	9.3	8.9	9.3	8.9	9.1	9.3	9.3	8.7	9.1	8.7	8.8	9.1	8.6
Deviation	Xi-T	0.1	0.2	0.3	-0.2	-0.5	0.1	0.1	-0.2	-0.2	-0.4	-0.1	0.3	0.3	-0.1	0.3	-0.1	0.1	0.3	0.3	-0.3	0.1	-0.3	-0.2	0.1	-0.4
CUSUM	Ci	0.1	0.3	0.6	0.4	-0.1	0.0	0.1	-0.1	-0.3	-0.7	-0.8	-0.5	-0.2	-0.3	0.0	-0.1	0.0	0.3	0.6	0.3	0.4	0.1	-0.1	0.0	-0.4
Moving Range	mR	*	0.1	0.1	0.5	0.3	0.6	0.0	0.3	0.0	0.2	0.3	0.4	0.0	0.4	0.4	0.4	0.2	0.2	0.0	0.6	0.4	0.4	0.1	0.3	0.5
Upper Mask		3.6	3.4	3.3	3.2	3.1	2.9	2.8	2.7	2.6	2.4	2.3	2.2	2.1	2.0	1.8	1.7	1.6	1.5	1.3	1.2	1.1	1.0	0.8	0.7	0.6
Lower Mask		-4.4	-4.2	-4.1	-4.0	-3.9	-3.7	-3.6	-3.5	-3.4	-3.2	-3.1	-3.0	-2.9	-2.8	-2.6	-2.5	-2.4	-2.3	-2.1	-2.0	-1.9	-1.8	-1.6	-1.5	-1.4

Plots of both units produced per week, and the cumulative sum (CUSUM) in the fund are shown in Figure 1.

Important properties of the cumulative sum plot are apparent from this example. When production is consistently on target the CUSUM plot is horizontal, when production is consistently above target the plot has a positive slope and when production is consistently below target the plot has a negative slope. In addition the mean production level over a period is simply the target production level plus the slope of the CUSUM plot.

### Monitoring a process with a target

Consider now a manufacturing process for lubricating oil where the target viscosity is 9.0 Centistokes (CSt) at 100°C (one CSt is one squared millimeter per second; water at 20 degrees C has a viscosity of about one CSt). Viscosity measurements are made on the output at 15-minute intervals during a long production run.

The first 25 measurements are shown in Table 2. Also shown are the successive deviations from target, the CUSUM values, i.e. the cumulative sum of these deviations from target, and moving range (mR) values that will be referred to later. As with the trivial example in the introduction, plots of both the Viscosity and the CUSUM are shown in Figure 2.

The run chart may be transformed into a Shewhart control chart for individual measurements by placing a center line at the position of the mean of the 25 measurements and upper and lower chart limits at the mean plus and minus three standard deviations respectively. It is common practice to estimate standard deviation from the mean moving range of span 2. Division of the mean moving range by 1.128, Hartley's constant for sample size 2, yields an estimate of standard deviation to be  $0.279/1.128 = 0.247$ . Hence the chart

limits are  $8.984 - 3 \times 0.247 = 8.24$  and  $8.984 + 3 \times 0.247 = 9.73$ . The chart is shown in Figure 3.

All the plotted points lie between the chart limits. The interpretation is that Viscosity is behaving in a stable, predictable manner with only **common cause** variation present. With the CUSUM chart a V-mask, displayed in Figure 4, may be used to investigate whether or not only common cause variation is present.

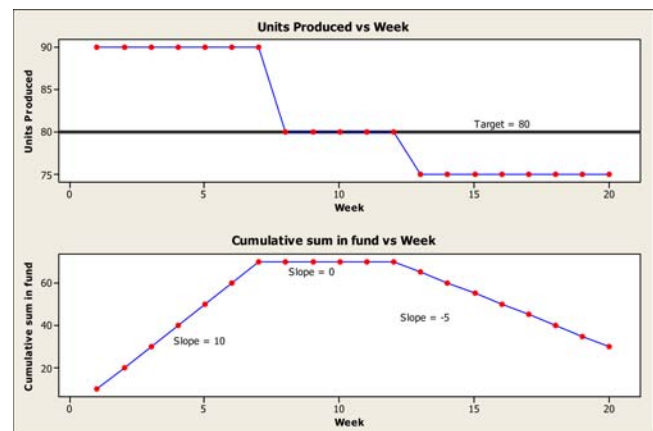


Figure 1.

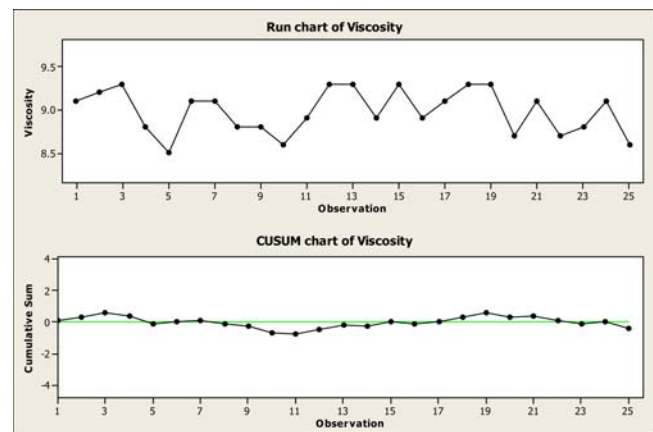


Figure 2.

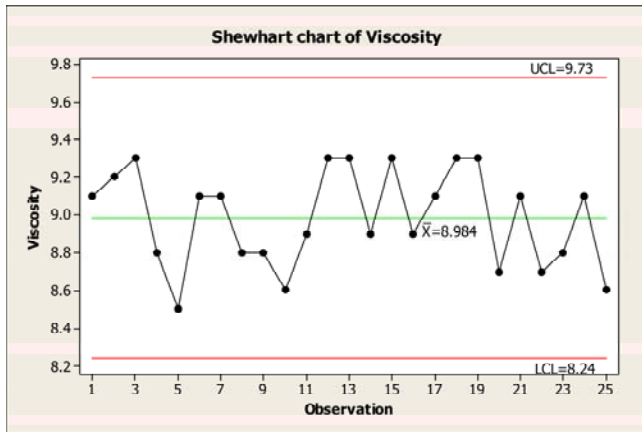


Figure 3.

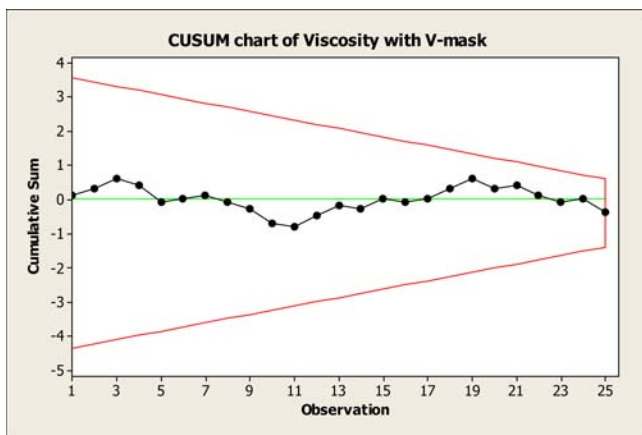


Figure 4.

All the plotted points lie within the arms of the V-mask. Again the interpretation is that Viscosity is behaving in a stable, predictable manner with only **common cause** variation present i.e. in what is frequently referred to as a state of statistical control or in-control state.

Construction of the mask, strictly a truncated V, is illustrated in Figure 5. (Details of the creation of the chart and V-mask using Microsoft Excel are provided in Appendix 1 and the associated formulae are given in Appendix 2.)

C represents the most recent CUSUM plotted. The vertical line segments AC and CB are each of length  $h\hat{\sigma}$  and the slopes of the arms are  $-k\hat{\sigma}$  (upper) and  $k\hat{\sigma}$  (lower) where  $\hat{\sigma}$  is the estimated standard deviation. The parameters  $h$  and  $k$  were assigned the widely-used values of 4.0 and 0.5 in this example. With no evidence of the presence of special cause variation from either chart they could both be employed for further monitoring of the process with the limits and mask respectively calculated from the initial 25 observations of Viscosity. As each new CUSUM value is plotted the mask is moved so that the mid-point of AB lies on the most recent point.

It should be noted that the mean, 8.984, of the 25 observations is close to the target viscosity of 9.0 CSt.

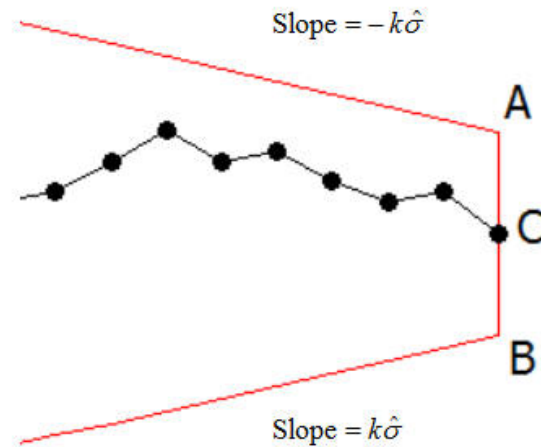


Figure 5.

With a further 10 observations to hand the charts were as shown in Figures 6 and 7. (The additional data are provided in the Worksheet Viscosity Data in the Microsoft Excel Workbook Data.xls.) Unless a macro is used, updating the CUSUM chart in Excel is time-consuming. With Release 15 of the Minitab software it is a simple matter to create a CUSUM chart that will update automatically as additional observations are added to the Worksheet containing the data.

Neither chart provides evidence of special cause variation. However on plotting the 49<sup>th</sup> data point the charts were as shown in Figures 8 and 9.

The occurrence of points outwith the arms of the V-mask is a signal from the CUSUM chart that provides evidence of **special cause** variation affecting the process. This might have been suspected from the change in the appearance of the CUSUM chart from a generally horizontal drift to a downward drift. In addition to providing evidence of a change in the process mean the CUSUM chart enables estimation to be made of: -

- (a) when the change occurred,
- (b) the magnitude of the change.

The first point outside the arms of the mask is (38, -0.5) and the final plotted point is (49, -3.0). The slope of the join is -0.23. The target of 9 added to this slope value is 8.77. Thus it is estimated that a change in the process mean occurred around the time that the 38<sup>th</sup> measurement was made and that the current mean is around 8.77. Such information is of value to those running the process. Typically in an industrial context steps would be taken to adjust the process so that it reverted to target.

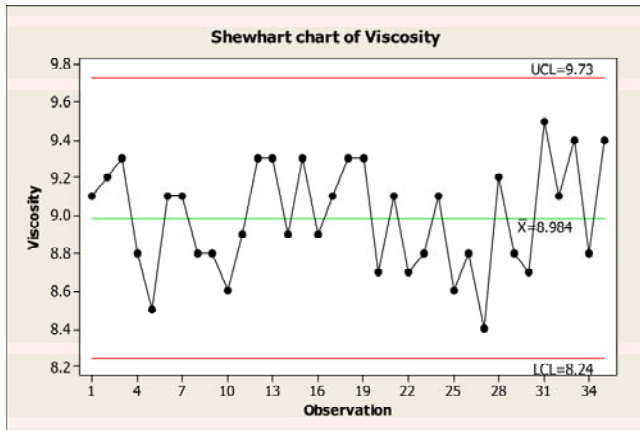


Figure 6.

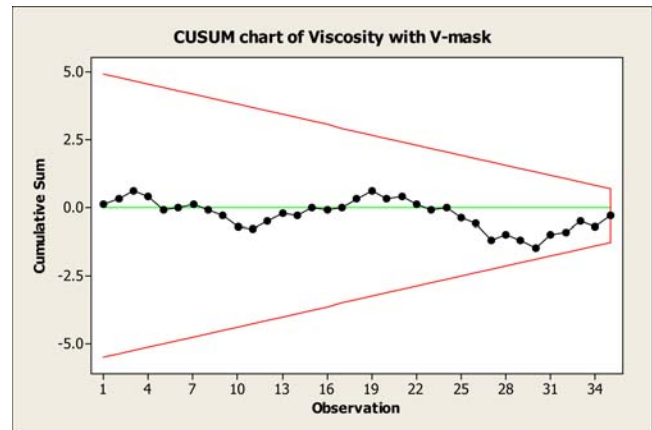


Figure 7.

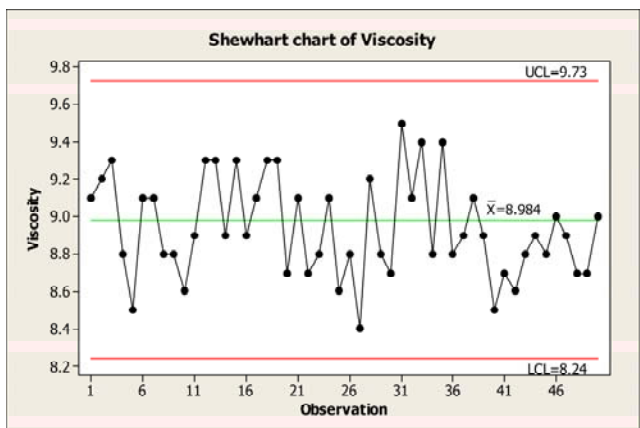


Figure 8.

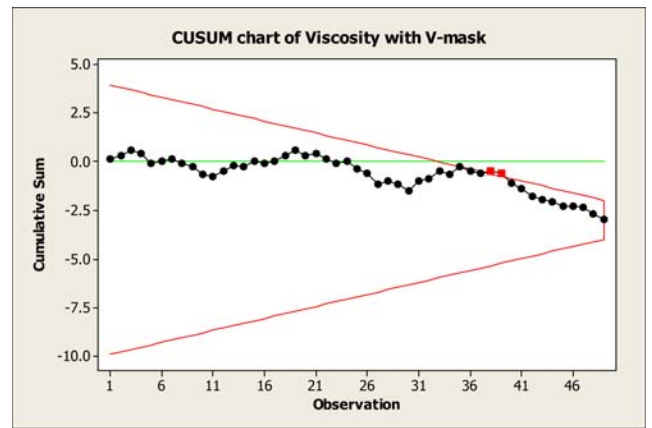


Figure 9.

Consider the scenario in which the oil production process is operating in a stable and predictable manner, i.e. in a state of statistical control with only common cause variation present, yielding oil such that the Viscosity measurements are normally distributed with mean 9.0 and standard deviation 0.1. Given that the parameters of the distribution are known then the probability that a point plotted on the Shewhart chart for Viscosity lies between the three sigma chart limits is 0.9973. The probability that a point lies outwith the limits is therefore 0.0027. Thus the probability of a false alarm signal of special cause variation arising is 0.0027 which is approximately 1 in 370 (the occurrence of a false alarm is analogous to committing a Type I error in hypothesis testing). Thus the mean number of points between false alarm signals will be 370. This is known as average run length, ARL, a concept that enables performance of control charts to be assessed and compared. For the CUSUM chart with  $h = 4$  and  $k = 0.5$ , the default parameters employed in Minitab, the corresponding in-control ARL is 335. One could change the in-control ARL for the Shewhart chart simply by changing the number of standard deviations separating the chart limits

from the center line from three but Shewhart (1931) argued from experience that three was “an acceptable economic value”. The in-control ARL for the CUSUM chart may be changed by altering the values of  $h$  and  $k$  employed in constructing the mask. Hawkins and Olwell (1995) provide a table giving the CUSUM in-control ARL as a function of  $h$  and  $k$ .

Were the process mean to shift by one standard deviation to 9.1, with no associated change in process variability, then the expected number of points plotted to yield a signal of special cause variation on the Shewhart chart is 44 i.e. the ARL is 44 for a one standard deviation shift in the process mean. However with the CUSUM chart the ARL is only 8. Were the process mean to change by four standard deviations then the ARL for the Shewhart chart would be 1.2 compared to 1.7 for the CUSUM chart. Thus the CUSUM chart is superior to the Shewhart chart in the detection of small changes in process mean performance while the opposite is the case for large changes. In situations where observations on a process are made at regular intervals the ARL following a process change links directly to time to occurrence of a signal of

evidence from the chart of that change having taken place.

Failure of a control chart to detect a process change when one has in fact occurred is analogous to committing a Type II Error in hypothesis testing. Hawkins and Olwell (1995) state that when a process being monitored using control charts is subject only to common cause variation “we would like the runs between the inevitable false alarms to be as long as possible” and that “if there has been a shift big enough to have practical implications, you would like to detect it as soon as possible”. The objectives of high in-control ARL coupled with low ARL when a shift of a magnitude that it is desirable to detect has occurred conflict. Making trade-offs between the two is analogous to the trade-offs made between both Type I and Type II Errors in hypothesis testing. They discuss design of a CUSUM chart in a situation where it is desirable to detect a shift of magnitude  $\Delta$  standard deviations which involves choosing  $k = 0.5\Delta$  and then using tables or software to select a value of  $h$  giving an acceptable in-control ARL.

The fundamental test for evidence, from a Shewhart control, of special cause variation affecting a process is the occurrence of a point more than three standard deviations from the center line. Additional tests have been devised and the eight tests provided in the Minitab software are as follows. The first four are referred to as the Western Electric Company (WECO) rules.

1. 1 point more than three standard deviations from center line
2. 9 consecutive points on same side of center line
3. 2 out of 3 consecutive points more than two standard deviations from center line (same side)
4. 4 out of 5 consecutive points more than one standard deviation from center line (same side)
5. 6 consecutive points, all increasing or all decreasing
6. 14 consecutive points, alternating up and down
7. 15 consecutive points within one standard deviation of center line (either side)
8. 8 consecutive points more than one standard deviation from center line (either side)

None of these tests signaled evidence of special cause variation for the Viscosity data considered above.

While the employment of additional tests reduces the ARL for the Shewhart chart in the detection of a change

in the process mean the price paid for the extra sensitivity is a reduction in the in-control ARL i.e. an increase in the number of false alarm signals. With 8 in place of 9 in the second test, employment of the first four tests reduces the in-control ARL from 370 to 92, thus quadrupling the number of false alarm signals of evidence of the presence of special cause variation for in-control processes.

### Monitoring a process with no target

Retrospective analysis of process performance data via CUSUM charting is referred to as post-mortem CUSUM charting by Roland Caulcutt (1995), but the term post-hoc CUSUM charting is used here (a report by one of the authors that referred to post-mortem CUSUM charts led a manager to believe initially that they displayed data on deceased patients!). In the above example there was a specific target for Viscosity but in situations where there is no specific target for a process performance measure of interest the performance of the process over a period may be investigated via CUSUM charting by using as an artificial target the mean of the observations over that period.

### Case I – Administration of Aspirin to ischaemic stroke patients

In the treatment of ischaemic stroke patients the prescription of the drug aspirin is frequent and the monthly proportion of ischaemic stroke patients administered aspirin within two days of admission to hospital is a performance indicator used in Scottish hospitals. Table 3 gives the data for a Scottish hospital for the 27-month period commencing with October 2006.

The mean of the monthly proportions is 67.0 so the CUSUM chart in Figure 10 was created with this value used as an artificial target.

Scrutiny of the chart suggests two distinct phases:

- one up to the end of 2007 with a downward slope which would indicate a mean below the artificial target value of 67.0
- a second consisting of the months of 2008 with an upward slope which would indicate a mean above the artificial target value of 67.0.

It should be noted that the final CUSUM value in this context is always zero.

Table 3

Month	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
%Asp $\leq$ 2	50	53	53	58	58	75	67	55	69	73	41	64	56	67	46	74	73	70	81	75	59	79	88	89	61	83	91



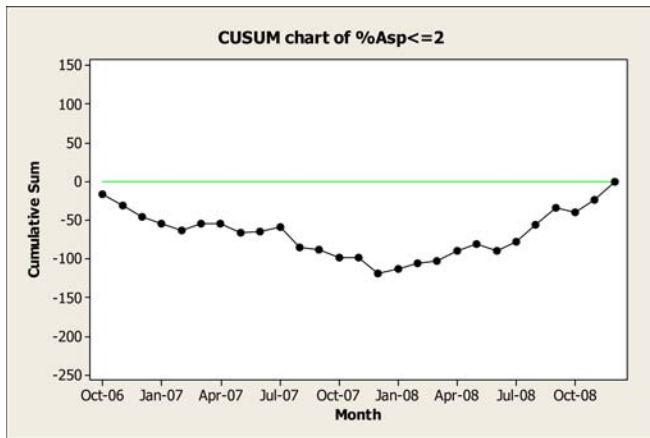


Figure 10.

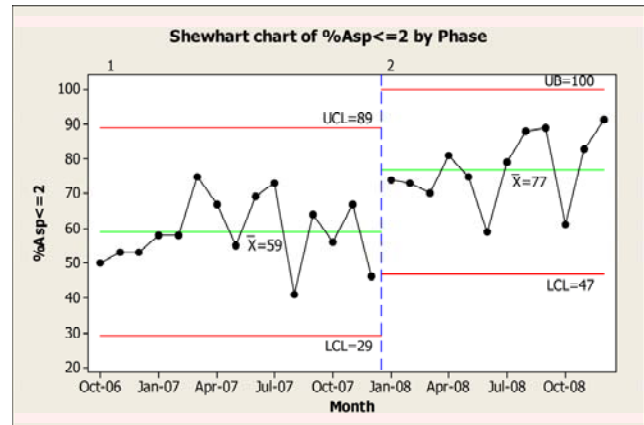


Figure 12.

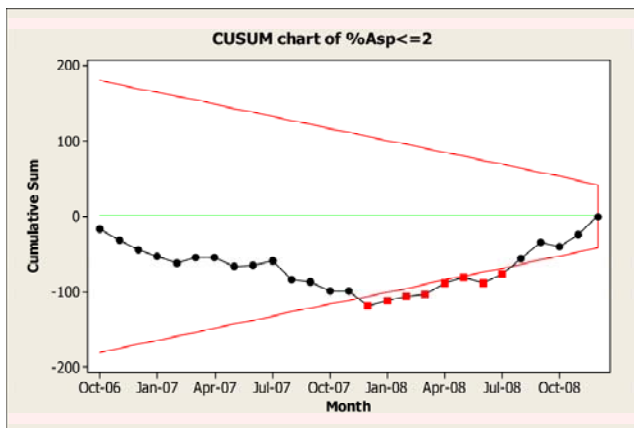


Figure 11.

Formal evidence of a process change is provided by the V-mask in Figure 11. Subsequently the Shewhart individuals charts displayed in Figure 12 were created for the two phases. Thus a change from a process displaying only common cause variation around a monthly mean level of around 60% to one displaying only common cause variation around a monthly mean approaching 80% is indicated. As the formula for the upper chart limit in the second phase yields a value in excess of 100% an upper bound (UB) is displayed on the chart instead. The improvement may be attributable to the establishment of a stroke outreach nurse service to the patient assessment area in the hospital and to faster communication of computed tomography brain scan results to clinical staff.

### Case II – PCS-12 scores of patients undergoing joint replacement surgery

The PCS-12 score is a summary measurement of physical health status computed from patient responses to the SF-12 generic quality-of-life questionnaire. Table 4 gives the mean increase in PCS-12 scores, one year after surgery, for patients at a Scottish hospital with hip/knee joint replacement operations.

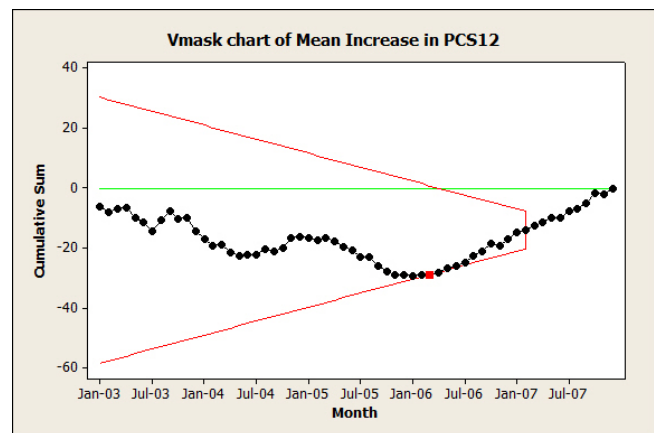


Figure 13.

The chart in Figure 13 provides evidence of a change having taken place around the beginning of 2006 – a minor change, from a clinical viewpoint, of two PCS-12 points on average, as indicated in Figure 14, but nonetheless an indication of improvement. No specific changes in surgical technique were introduced at the beginning of 2006; however care delivery was re-organized with patients becoming involved in the planning of their recovery programs and the clinical director for orthopedics believes that this has led to the change.

### Case III – Numbers of hospital-acquired MRSA infections

Hospitals in the UK are striving to reduce hospital-acquired methicillin resistant *Staphylococcus aureus* (MRSA) infection of patients. Table 5 gives the monthly numbers of incidences at a group of Scottish hospitals over a 30-month period.

Table 4.

Month in 2003	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Patients	20	17	49	45	38	50	48	42	33	30	49	43
Mean increase in PCS12	4.43	8.62	11.30	11.14	6.98	8.85	7.67	14.01	13.72	7.89	10.67	5.93
Month in 2004	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Patients	46	37	54	53	57	77	54	42	35	48	41	53
Mean increase in PCS12	8.07	8.05	11.05	7.81	9.32	10.88	10.37	12.14	9.78	11.85	13.51	11.15
Month in 2005	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Patients	55	72	68	86	82	92	58	68	74	50	53	36
Mean increase in PCS12	9.81	9.79	11.43	9.04	8.66	9.62	8.24	10.43	7.32	8.46	9.51	10.56
Month in 2006	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Patients	49	55	68	57	62	60	87	74	64	91	74	66
Mean increase in PCS12	10.00	10.69	10.44	11.57	11.96	10.98	11.78	12.48	12.20	13.07	9.50	12.79
Month in 2007	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Patients	79	103	106	79	93	67	74	84	67	90	95	61
Mean increase in PCS12	12.75	11.20	12.01	11.63	11.96	10.36	12.62	11.29	12.36	14.05	10.15	12.29

Table 5.

	2007												2008		
Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar
MRSA	11	20	15	19	13	15	13	13	20	11	9	11	5	17	10
	2008												2009		
Month	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June
MRSA	13	15	12	18	15	16	12	5	14	14	9	7	6	8	5

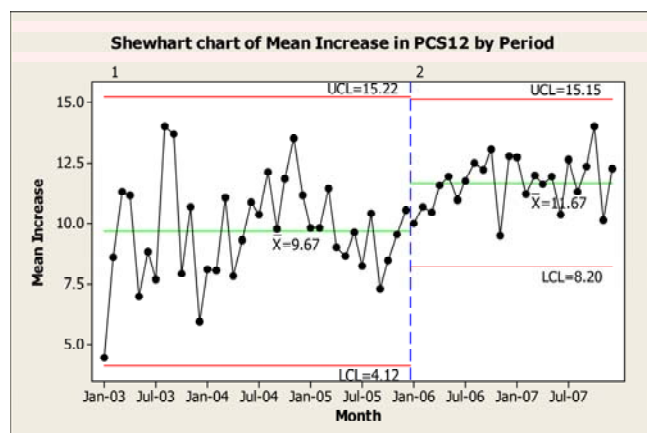


Figure 14.

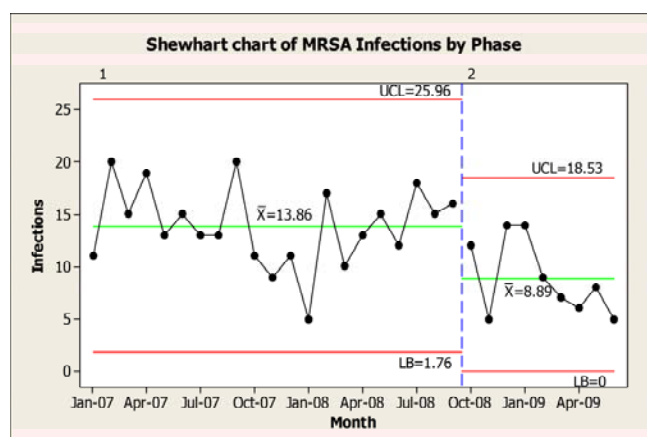


Figure 16.

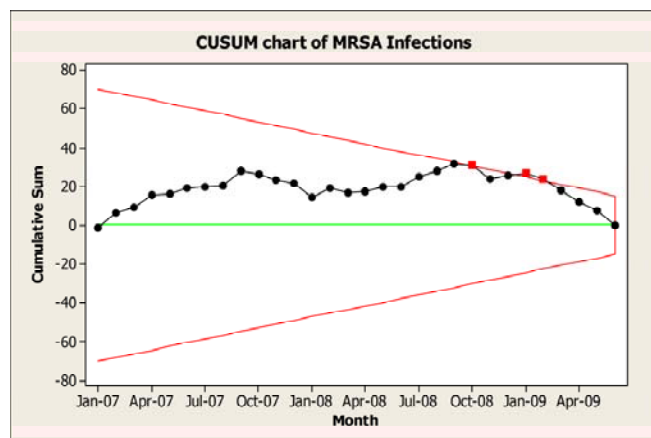


Figure 15.

The CUSUM chart in Figure 15 provides evidence of a change having occurred around September 2008.

The Individuals charts in Figure 16 indicate a reduction in the number of incidences of MRSA infections from a mean of around 14 per month to a mean of around 9 per month. Since late 2007 steps have been taken within the group of hospitals to improve hand hygiene and to introduce a patient safety program that employs care bundles - sets of interventions that may be applied during the insertion and management of vascular catheters, the commonest source of MRSA infections. Thus the charting provides evidence that measures taken in the hospitals to reduce the incidence of infections have been effective.



## Discussion

The majority of the charts in this paper were created using Release 15 of the statistical software package Minitab. Details of the use of Minitab and of its facilities for the creation of Shewhart charts may be found in Henderson (2006). Other software packages may be used e.g. CHARTrunner, JMP, Quality Analyst, Statgraphics etc. The charts could be created using spreadsheets such as Microsoft Excel or by using Excel add-in software such as QI Macros, SQC for Excel etc.

The data are provided in the Worksheets Aspirin, PCS-12 and MRSA in the Microsoft Excel Workbook Data.xls. Also provided are Worksheets giving the artificial data set, the Viscosity data, the computations for the initial Viscosity CUSUM chart and V-mask, together with the CUSUM chart.

The three data sets considered involved proportions in the case of the stroke care data, scores that may be considered equivalent to measurements in the case of the surgical data and counts in the case of the infections data. Only Shewhart charts intended for individual measurements were employed and some may question the use of this type of chart with proportions and counts when specialised Shewhart charts for such data are available. In referring to the use of the Shewhart chart for individual values Wheeler and Poling (1998) point out "it gives limits that are empirical" whereas specialist Shewhart charts for proportions and counts, based on the binomial and Poisson distributions respectively, use "a specific probability model to construct theoretical limits". They warn that "if the theoretical model does not correctly describe the data, then the theoretical limits will be incorrect, while the empirical limits will still be appropriate". In the course of their work in healthcare the authors have sought to promote the use control charting. One of the merits of the Shewhart chart for individual values is that in essence chart limits at mean  $\pm 3$  standard deviations are analogous to clinical use of reference intervals e.g. for patient haemoglobin levels. Typically the reference interval is computed as mean  $\pm 2$  standard deviations for a variable that is normally distributed. Concerns about the applicability of the theoretical models in the case of the stroke care data and the infections data, coupled with the relative simplicity of the Shewhart chart for individual values, led the authors to opt for its use in the investigations reported. Investigation of the data in other formats using other Shewhart charts is desirable in the future.

In addition to its superiority over the Shewhart chart for the detection of small changes in the process mean the

CUSUM chart has a single test for the presence of evidence of special cause variation i.e. a point outwith the arms in the V-mask form and a point outwith the chart limits in the tabular (algorithmic) form. In contrast the Shewhart chart has associated with it a plethora of tests. New users of control charts often have difficulty in deciding which tests to employ.

Hypothesis testing may be used to determine whether or not an improvement in performance has resulted from a process change. Some argue that process monitoring using control charts is equivalent to continuous hypothesis testing. In this context William Woodall (2000) believes that "it is very important to distinguish between use of a control chart on a set of historical data to determine whether or not a process has been in statistical control (Phase 1) and its use prospectively with samples taken sequentially over time to detect changes from an in-control process (Phase 2)". He states that in Phase 1 scenarios the application of control charting is akin to exploratory data analysis whereas in Phase 2 scenarios "it does closely resemble repeated hypothesis testing". Woodall (2006) also discusses applications of control charts in healthcare and public health surveillance and provides nearly 150 references. The investigations reported here are in the exploratory Phase 1 category. In the future we envisage adoption of prospective Phase 2 applications e.g. managers of the Scottish Stroke Care Audit, on which one of the authors is employed, have asked hospitals caring for stroke patients to commence entering data prospectively in 2010 with the aim of providing stroke clinicians with faster feed-back on performance than is currently provided.

Details of CUSUM charts for sample means and discrete data may be found in the book by Hawkins and Olwell (1995) and its website provides programs and Excel templates for CUSUM charting. Binary outcome CUSUM charts are being applied in healthcare. Steiner et al (2000) developed a CUSUM monitoring system where the binary outcome considered was that of a surgical procedure (success / failure). The system incorporated risk adjustment to take into account the pre-operative risk of failure for each patient. Tennant et al (2007) report a systematic review of the uses control charts to monitor clinical variables in individual patients with conclusion that "control charts appear to have a promising but largely under-researched role in monitoring clinical variables in individual patients". They also argue that rigorous evaluation of the application of control charts in healthcare is required.

Douglas Montgomery (2009) and others advocate the use of the tabular or algorithmic form of the CUSUM chart. Hawkins and Olwell (2000) refer to it as the decision interval form. The chart that results is similar in appearance to a Shewhart chart. Setting up the tabular form with a particular choice of  $h$  and  $k$  is algebraically equivalent to setting up the V-mask form with the same values for  $h$  and  $k$ . It involves calculation and plotting of both an upper CUSUM and a lower CUSUM. Evidence of an upward shift in the process mean is provided by the occurrence of an upper CUSUM value greater than the upper chart limit; evidence of a downward shift in the process mean is provided by the occurrence of a lower CUSUM value less than the lower chart limit. Figure 17 shows the Viscosity data considered earlier displayed in both types of chart created using Minitab.

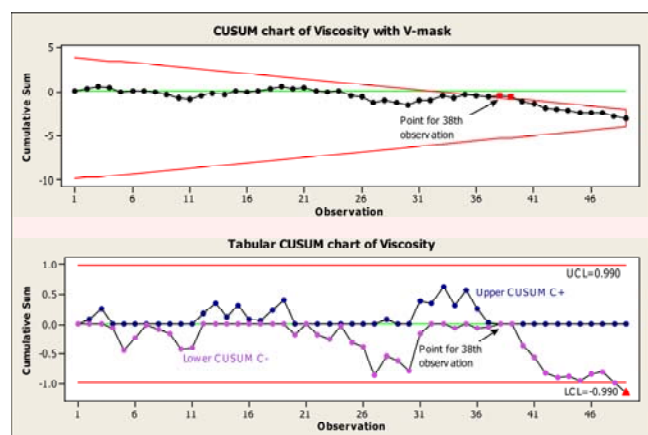


Figure 17.

In both cases the signal providing evidence of special cause variation arises on plotting the data from the 49th observation and in both cases the charts suggest that the change took place around the time that the 38th observation was made. Further details and the formulae for computing the upper and lower CUSUM values and the positions of the chart limits are given in Hawkins and Olwell (1995) and in Montgomery (2009). The formulae required are also provided in Appendix 2. However the authors believe that even the visual scrutiny of the basic form of the CUSUM chart considered in this paper can be a precursor to valuable insights into time-ordered data sets.

In conclusion the authors have found that, having detected evidence of changes in the level of performance of healthcare processes using post-hoc CUSUM charting, Shewhart charting, with stages, provides a good vehicle for communicating that evidence to clinical and managerial staff, generally unfamiliar with CUSUM charts. Thus the post-hoc CUSUM approach, with use of the mean performance over the time period of interest as

an artificial target, provides a useful tool for exploratory retrospective analysis of healthcare data. The only reference the authors have found to the use of the overall mean as an artificial target in order to be able to employ CUSUM methods is in Caulcutt (1995). The authors believe that this technique removes a barrier to potential applications of CUSUM charts in healthcare.

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## Appendix 1 – Creation of a CUSUM chart with V-mask in Microsoft Excel

A description is given of the creation in Microsoft Excel of the CUSUM chart and mask for the viscosity data in Table 2.

	A	B	C	D	E
1	Observation	i	1	2	3
2	Viscosity	$X_i$	9.1	9.2	9.3
3	Deviation	$X_i - T$	=C2-9	=D2-9	=E2-9
4	CUSUM	$C_i$	=C3	=C4+D3	=D4+E3
5	Moving Range	mR	*	=ABS(D2-C2)	=ABS(E2-D2)
6	Upper Mask		=D6+\$AC\$9*\$AC\$5	=E6+\$AC\$9*\$AC\$5	=F6+\$AC\$9*\$AC\$5
7	Lower Mask		=D7-\$AC\$9*\$AC\$5	=E7-\$AC\$9*\$AC\$5	=F7-\$AC\$9*\$AC\$5

Figure 18.

Having entered the formulae for calculating the first two CUSUM values in cells C4 and D4 as shown in Figure 18, the formula in D4 may be copied by dragging to the right into all the other necessary cells in row 4. Note that there is no moving range value available with only one observation to hand – hence the asterisk in cell C5.

Y	Z	AA	AB	AC
23	24	25	Mean	
8.8	9.1	8.6	=SUM(C2:AA2)/25	
=Y2-9	=Z2-9	=AA2-9		
=X4+Y3	=Y4+Z3	=Z4+AA3	Mean mR	Sigma est.
=ABS(Y2-X2)	=ABS(Z2-Y2)	=ABS(AA2-Z2)	=SUM(D5:AA5)/24	=AB5/1.128
=Z6+\$AC\$9*\$AC\$5	=AA6+\$AC\$9*\$AC\$5	=AA4+AB9*AC5		
=Z7-\$AC\$9*\$AC\$5	=AA7-\$AC\$9*\$AC\$5	=AA4-AB9*AC5		
			h	k
			4	0.5

Figure 19.

Having computed the Mean, Mean moving range (Mean mR), Standard deviation estimate (Sigma est.) and having entered values for h and k in cells AB9 and AC9 both the formulae in cells AA6 and AA7 and the formulae in cells Z6 and Z7 were created as displayed in Figure 19. The remaining formulae required in rows 6 and 7 were created by copying and dragging to the left the formulae in cells Z6 and Z7. Use of Line Chart gave the CUSUM chart displayed in Figure 20.

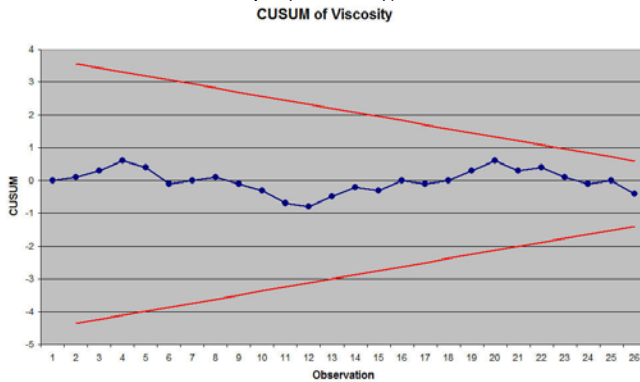


Figure 20.

## Appendix 1 – Creation of a CUSUM chart with V-mask in Microsoft Excel

### Basic Form

The chart consists of a plot of  $C_i$  versus  $i$  where:

$$C_i = (x_i - T) + C_{i-1},$$

$i$  is the observation number for  $i = 1, 2, 3, \dots$

$x_i$  is the  $i$ th observation

$C_i$  is the  $i$ th CUSUM

$T$  is the target value

$C_0$  is defined to be 0

$V$  – mask arm slopes are  $\pm k\hat{\sigma}$

Mask width at the narrowest point is  $2h\hat{\sigma}$

$\hat{\sigma}$  is the standard deviation estimate obtained by

dividing the mean moving range of span 2 by 1.128

### Tabular Form

The chart consists of a plot of both  $C_i^+$  and  $C_i^-$  versus  $i$  where:

$$C_i^+ = \max(0, C_{i-1}^+ + x_i - [T + k\hat{\sigma}])$$

$$C_i^- = \min(0, C_{i-1}^- + x_i - [T + k\hat{\sigma}])$$

$i$  is the observation number for  $i = 1, 2, 3, \dots$

$x_i$  is the  $i$ th observation

$C_i^+$  is the  $i$ th upper CUSUM

$C_i^-$  is the  $i$ th lower CUSUM

$T$  is the target value

Both  $C_0^+$  and  $C_0^-$  are defined to be 0

$V$  – mask arm slopes are  $\pm k\hat{\sigma}$

The chart limits are located at  $\pm h\hat{\sigma}$

$\hat{\sigma}$  is the standard deviation estimate obtained by

dividing the mean moving range of span 2 by 1.128

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